Sanda Bujacic, Department of Mathematics, University of Zagreb, Croatia

Linear Forms in Logarithms (abstract)

Four Faces of Number Theory, Würzburg, July, 2014.

Hilbert's problems form a list of twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. Those problems were all unsolved at the time, and several of them were very influential for 20th century mathematics. David Hilbert believed that mathematicians have to find new machineries and methods to be able to solve those problems.

The seventh Hilbert problem was dealing with the transcendence of α^β for algebraic $\alpha \neq 0, 1$ and irrational algebraic β. This problem was solved separately by Gelfond and Schneider. They have proved that, if $\alpha_1, \alpha_2 \neq 0$ are algebraic numbers such that $\log \alpha_1, \log \alpha_2$ are linearly independent over $\mathbb{Q}$, then

$$\beta_1 \log \alpha_1 + \beta_2 \log \alpha_2 \neq 0,$$

β_1, β_2 are algebraic numbers.

In 1935 Gelfond found a lower bound for the absolute value of the linear form

$$\Lambda = \beta_1 \log \alpha_1 + \beta_2 \log \alpha_2 \neq 0.$$

He proved that

$$\log |\Lambda| >> -h(\Lambda)^\kappa,$$

where $h(\Lambda)$ is logarithmic height of the linear form and $\kappa > 5$. Gelfond also noticed that generalization of his results could prove a huge amount of unsolved problems in number theory. In 1966 A. Baker in his article "Linear forms in logarithms of algebraic numbers I, II, III", gave an effective lower bound on the absolute value of a nonzero linear form in logarithms of algebraic numbers, that is for a nonzero expression of the form

$$\sum_{i=1}^{n} = b_i \log \alpha_i,$$

where $\alpha_1, \ldots \alpha_n$ are algebraic numbers and $b_1, \ldots b_n$ are integers.

In my lectures, I shall be dealing with basic definitions of those concepts and I shall introduce most important theorems and proofs. Also, I shall introduce some Baker type inequalities available today which are easy to apply. In order to illustrate this very important machinery I shall introduce some examples. I shall show that the largest Fibonacci number having only one repeated digit in its decimal expression is 55 and I will show that $d = 120$ is the largest positive integer such that $\{d+1, 3d+1, 8d+1\}$ are all three perfect squares.