Poisson Geometry and Normal Forms: A Guided Tour through Examples

Eva Miranda

Summer School 2015

This minicourse aims to cover basic material in Poisson Geometry with a special focus on examples. There are no special prerequisites to follow this minicourse except for basic differential geometry.

Poisson manifolds constitute a natural generalization of Symplectic manifolds. Many problems in mechanics turn out to be naturally formulated in the Poisson language. In contrast to symplectic manifolds where there are no local invariants (Darboux), Poisson manifolds do present local invariants. The aim of the minicourse is to present an introduction to Poisson Geometry and the study of normal forms (how the structures look like locally). We also plan to explain some of the problems considered in their semilocal and global study and introduce the study of symmetries (group actions) in these manifolds. We will end up the minicourse presenting an application of the study of group actions to integrable systems (stressing the particular case of b-Poisson manifolds). The minicourse will consist of 5 sessions.

i.) Lecture 1: Motivation, definition and examples.

ii.) Lecture 2: Local Poisson Geometry: Splitting theorem and normal forms.

iii.) Lecture 3: Going global: Poisson cohomology: Examples.

iv.) Lectures 4 and 5: An application: Integrable Systems and Hamiltonian actions. The case of b-Poisson manifolds.

References

[1] Dufour, J.-P., Zung, N. T.: *Poisson Structures and Their Normal Forms*, vol. 242 in *Progress in Mathematics*. Birkhäuser Verlag, Basel, Boston, New York, 2005.

[2] Guillemin, V., Miranda, E., Pires, A. R.: *Codimension one symplectic foliations and regular Poisson structures*. Bull. Braz. Math. Soc. (N.S.) **42**.4 (2011), 607–623.

[3] Kiesenhofer, A., Miranda, E., Scott, G.: *Action-angle variables and a KAM theorem for b-Poisson manifolds*. Preprint **arXiv:1502.03489** (2015). Accepted at Journal de Mathématiques Pures et Appliquées.

[4] Laurent-Gengoux, C., Miranda, E., Vanhaecke, P.: *Action-angle coordinates for integrable systems on Poisson manifolds*. Int. Math. Res. Not. IMRN .8 (2011), 1839–1869.

[5] Laurent-Gengoux, C., Pichereau, A., Vanhaecke, P.: *Poisson Structures*, vol. 347 in *Grundlehren der mathematischen Wissenschaften*. Springer-Verlag, Heidelberg, Berlin, New York, 2013.

[6] Weinstein, A.: *The Local Structure of Poisson Manifolds.* J. Diff. Geom. **18** (1983), 523–557.