Algebraic Aspects of Deformation Quantization

Martin Bordemann

Summer School 2015

First I deal with graded K-modules (where K is a commutative associative unital ring containing the rationals, and I shall advertise some 'algebraic differential geometry), their tensor products, sign rules , and suspensions, and give some categorical benedictions (coherence theorems for enriched categories) to justify sloppy sign use. Then graded algebras, free algebras (and their corresponding coalgebras) are recalled, and, more importantly, their corresponding CCC (counital coaugmented connected) coalgebras will then be presented. The induced morphisms and coderivations will be constructed by convolution, one of the main technical tools. A by-product will be the proof of the pre-Lie-identity for the Gerstenhaber bracket which is important in the deformation theory of associative algebras, and gives a proof for the properties of the Hochschild differential. After this I recall graded CCC symmetric (co)algebras and the more simple convolution formulas (due e.g. to Helmstetter) for morphisms and (co)derivations, yielding the Nijenhuis-Richardson bracket. We present the Serre functor associating pointed differentiable manifold to CCC cocommutative coassociative coalgebras which is algebraicized by Kontsevich to the graded case. We then define L-infinity algebras as graded coderivations of square zero on free graded cocommutative CCC coalgebras, their morphisms and (quasi)isomorphisms, their Maurer-Cartan elements and moduli spaces, and show how the existence and uniqueness-up-to-equivalence problem in deformation quantization is solved (by Kontsevich) by constructing a so-called formality map, i.e. a quasi-isomorphism.

In case there is some time, I can comment on universal envelopping algebras for Lie algebras over K, and their PBW-theorem, and on a (generalization) of the Voronov L_∞ structures and some possible applications.