Poster Session

2014 Winter School Würzburg

1 Tuesday session

Traveling waves for a bistable equation with nonlocal-diffusion
Franz Achleitner (TU Wien, Austria)

We consider a single component reaction-diffusion equation in one dimension with bistable nonlinearity and a nonlocal space-fractional diffusion operator of Riesz-Feller type. Our main result shows the existence, uniqueness (up to translations) and stability of a traveling wave solution connecting two stable homogeneous steady states. In particular, we provide an extension to classical results on traveling wave solutions involving local diffusion. This extension to evolution equations with Riesz-Feller operators requires several technical steps. These steps are based upon an integral representation for Riesz-Feller operators, a comparison principle, regularity theory for space-fractional diffusion equations, and control of the far-field behavior.

Relaxation results for some functionals depending on rational functions
Omar Boussaid (Chlef, Algeria)

A family of functionals depending on rational functions changing sign on the cone of rank one matrix is considered. By some suitable decomposition lemmas, we are able to explicitly compute the quasiconvex envelope of such functions. The quasiconvex hull guarantees the weak lower semi continuity for minimization problems in the Calculus of variations; see, e.g., [2], for the definition and characterization of such notion and other non-convex notions. Such problems have been intensively studied in the last thirty years and are motivated by several branches of science as nonlinear elasticity, optimal design, materials sciences and many others.

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New fractional results on differential systems and probability theory
Zoubir Dahmani (Mostaganem, Algeria)

In this poster, we present new results for the existence and uniqueness of solutions for nonlinear differential systems of arbitrary order. Some illustrative examples are also discussed. Other fractional integral results are also generated.

A low-volume fraction limit for austenite-martensite interfaces in shape-memory alloys
Johannes Diermeier (Bonn, Germany)

So called "shape-memory materials" have the property that they recover their shapes under heating if they have been deformed at a low temperature. The reason for this behaviour are different crystaline lattices that occur at low, respectively at high temperatures. One can observe the occurance of microstructure at interfaces between different phases of the material.
In order to understand those microstructures we consider a variational model of the energy of a deformation. We restrict ourselfes to a scalar valued, two dimensional, geometrically linearized case with two variants of martensite (low temperature state) in which one of the variants has a much smaller volume fraction then the other one. The energy is then given as

$$E^{\varepsilon,\theta}(v) = \int_\Omega \partial_x v^2 + \min\{|\partial_y v + \theta|^2, |\partial_y v - 1|^2\} \, \mathrm{d}\mathcal{L}^2 L + \varepsilon||D^2 v||(\Omega)$$

where $\theta \ll 1$ is the small volume fraction, $\Omega = (0,1)^2$ and v has zero-boundary values on the left edge of the square.
It is known that a twinning of the material gives the optimal energy scaling for $\varepsilon \leq C\theta^2$, while for $\varepsilon > c\theta^2$ the optimal scaling is realized by a single phase of martensite. We are interested in the behaviour between those two cases and therefore choose $\varepsilon = \sigma\theta^2$ for some σ and consider the reduced model by means of Γ-limit.

Modeling of Magnetic Complex Fluids
Johannes Forster (Würzburg, Germany)

We investigate magnetic fluids (ferrofluids) in the framework of complex fluids. From a continuum mechanical setting and an energetic ansatz for the material, we seek to derive PDEs to describe its behavior. We outline the process of modeling and also the energetic variational approach. Moreover, we introduce our model for the evolution of the magnetization of the material and highlight the problems that arise in the establishment of the PDEs.

A quantitative geometric rigidity result in SBD and the derivation of linearized models from nonlinear Griffith energies in fracture mechanics

Manuel Friedrich (Augsburg, Germany)

We derive Griffith functionals in the framework of linearized elasticity from nonlinear and frame indifferent energies via Gamma-convergence. The convergence is given in terms of rescaled configurations measuring the displacement of the deformations from piecewise rigid motions which are constant on each connected component of the cracked body. The key ingredient to establish a compactness result is a quantitative geometric rigidity result for special functions of bounded deformation (SBD). This estimate generalizes the result of Friesecke, James, Müller in nonlinear elasticity theory and the piecewise rigidiy result of Chambolle, Giacomini, Ponsiglione for SBV functions which do not store elastic energy. The results stated here are subject of ongoing work.

On asymptotic stability of nonlinear stochastic evolution equations

Mohamed Ali Hammami (Sfax, Tunisia)

In this work we establish some sufficient conditions ensuring almost sure asymptotic stability with a non-exponential decay rate for solutions in practical sense to stochastic evolution equations based on Lyapunov techniques ($[1]-[4]$). Furthermore, an illustrative numerical example is given. This is joint work with Tomás Caraballo (Sevilla, Spain) and Lassaad Mchiri (Sfax, Tunisia).

References

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Sobolev spaces on locally compact abelian groups

Tomasz Kostrzewa (TU Warsaw, Poland)

We research Sobolev spaces on locally compact abelian (LCA) groups. We focus on compactness embedding results, and we prove an analog for LCA groups of the classical Rellich lemma and of the Rellich-Konrachov compactness theorem. Furthermore we introduce Sobolev spaces on subsets of LCA groups and study its main properties, including the existence of compact embeddings into L^p-spaces.

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Asymptotic spectral analysis in quantum waveguides with heterogeneous fibers

Carolin Kreisbeck (Regensburg, Germany)

In recent years nanowires have become an active field of research in the physics community. We discuss two settings in which heterogeneous structures affect propagation through a quantum waveguide, highlighting their interaction with curvature and torsion.

First, we analyze the macroscopic behavior of a wire made of composite fibers with microscopically periodic texture, which amounts to determining the asymptotic behavior of the spectrum of an elliptic Dirichlet eigenvalue problem with finely oscillating coefficients in a three-dimensional tube with shrinking cross section. A suitable formal expansion suggests that the effective one-dimensional limit problem is of Sturm-Liouville type and yields the explicit formula for the underlying potential. In the torsion-free case, these findings are made rigorous by performing homogenization and 3d-1d dimension reduction for the two-scale problem in a variational framework by means of Γ-convergence.

Second, waveguides with non-oscillating inhomogeneities in the cross section are investigated. This leads to explicit criteria for propagation and localization of eigenmodes.

This is joint work with Luísa Mascarenhas.

Dynamics for a system of screw dislocations

Marco Morandotti (Instituto Superior Técnico, Portugal)

We describe the dynamics for a system of screw dislocations subject to anti-plane shear. Variational techniques allow us to find minimizers for the energy functional associated with the system of dislocations in an elastic medium. Building on a model due to Cermelli and Gurtin, a weak notion of solutions (in the sense of Filippov) to ordinary differential equations is used to solve the dynamics problem. Some examples of interesting scenarios complement the presentation.

Laplace equation in domains with an increasing crack: asymptotic development of the energy of the solutions

Gianluca Orlando (SISSA, Italy)

We consider the weak solution of the Laplace equation in a planar domain with a straight crack, prescribing a homogeneous Neumann condition on the crack and a nonhomogeneous Dirichlet condition on the rest of the boundary. We prove that the energy of the solution is a smooth function of the crack length and we determine its Taylor expansion starting from the well known asymptotic expansion of the solution around the crack tip. In particular, for every k we provide an algorithm which allows us to express the k-th derivative of the energy in terms of a finite number of coefficients of the asymptotic expansion of the solution and of a finite number of other parameters, which only depend on the shape of the domain.

Thermomechanically-coupled model for martensitic thin films on the mesoscopic scale

Gabriel Pathó

In the framework of generalized standard materials, we present a mesoscopic model for martensitic thin films, where thermomechanical effects are included. This particular model has been derived (mathematically rigorously, that is, in an ansatz-free setting) in two steps. First, we provide a dimension reduction procedure in a constitutively defined 3D model that considers microscopic effects as well. Then, the second step is a scale transition from micro to meso-scale (with the aid of so-called Young measures) when we neglect these microscopic effects.

Relaxation in Crystal Plasticity with Latent Hardening

Richard Schubert (Bonn, Germany)

We consider in the setting of calculus of variations the elasto-plastic deformation of a crystal under the nonconvex constraint of infinite latent hardening. That means that each material point can only deform in one of finitely many slipdirections leading to fine-scale oszillations between the different slip-phases. We consider the case of a linear plastic energy with a slight quadratic perturbation (the case of exactly linear energy was treated by S. Conti and M. Ortiz in "Dislocation Microstructures and the Effective Behaviour of Single Crystals") and compute the relaxed energy functional whose minimizer represents the macroscopic (relaxed) behaviour of the crystal. We then further characterize the relaxed local energy and show by an analysis of the rank-one directions that in two dimensions for arbitrary plastic energy the quasiconvex envelope equals the convex one. In three dimensions we obtain closeness of the quasiconex and convex envelope in terms of the perturbation.

Q-tensor continuum energies as limit of head-to-tail symmetric spin systems

Francesco Solombrino (TU München, Germany)

We consider a class of spin-type discrete systems and analyze their continuum limit as the lattice spacing goes to zero. Under standard coerciveness and growth assumptions together with an additional head-to-tail symmetry condition, we observe that this limit can be conveniently written as a functional in the space of Q-tensors. We further characterize the limit energy density in several cases (both in 2 and 3 dimensions). In the planar case we also develop a second-order theory and we derive gradient or concentration-type models according to the chosen scaling. This is a joint work with M. Cicalese (TU München) and A. Braides (University of Rome Tor Vergata).

Upscaling of Dislocation Walls in Finite Domains

Patrick van Meurs (TU Eindhoven, The Netherlands)

In this poster, I address a scientific challenge occurring in many complex systems: predicting the macroscale collective behaviour induced by interacting particles on the microscale. I do this by looking at the specific example of plasticity of metals.

Plastic deformation of metals is facilitated by the movement of impurities (defects) in the crystal lattice of the metal. These impurities are called dislocations. Each dislocation causes a stress field in the metal, and dislocations can move under shear stress. Dislocations in the metal interact, and since there are many dislocations per unit volume (say N), we are interested in passing to the continuum limit, i.e. $N \to \infty$.

Due to singular, non-local and orientation-dependent interactions between dislocations, this upscaling is quite challenging mathematically. Therefore, we examine a very specific configuration of a single type of dislocations (straight edge dislocations with parallel slip planes), which results in a 1-dimensional model. Recent results show that even in the stationary case, the upscaling is non-trivial and yields different interesting results (depending on the scaling of the parameters).

The poster presents our latest results about how this model can be extended to a finite domain. Our open problems are given by an outlook on how we can extend these results by making the model more realistic.

2 Thursday session

Nonlocal interaction equations with singular kernels: Wasserstein gradient flow vs entropy solutions

Giovanni Bonaschi (TU Eindhoven, The Netherlands; Pavia, Italy)

We prove the equivalence between the notion of Wasserstein gradient flow for a onedimensional nonlocal transport PDE with attractive/repulsive Newtonian potential on one side, and the notion of entropy solution of a Burgers-type scalar conservation law on the other. The solution of the former is obtained by spatially differentiating the solution of the latter. The proof uses an intermediate step, namely the L2 gradient flow of the pseudo-inverse distribution function of the gradient flow solution. We use this equivalence to provide a rigorous particle-system approximation to the Wasserstein gradient flow, avoiding the regularization effect due to the singularity in the repulsive kernel. The abstract particle method relies on the so-called wave-front-tracking algorithm for scalar conservation laws. Finally, we provide a characterization of the sub-differential of the functional involved in the Wasserstein gradient flow.

Local and global minimality results for a nonlocal isoperimetric problem on $\mathbb{R}^N$

Riccardo Cristoferi (SISSA, Italy)

We consider a nonlocal isoperimetric problem defined in the whole space $\mathbb{R}^N$, whose nonlocal part is given by a Riesz potential with exponent $\alpha \in (0, N-1)$. We show that critical configurations with positive second variation are local minimizers and satisfy a quantitative inequality with respect to the L^1 norm. We also determine explicitly the critical mass above which the ball is no longer a local minimizer. Finally we apply our local minimality criterion to deduce that for small masses the ball is also the unique global minimizer, and that for small exponents α in the non-local term the ball is the unique minimizer as long as the problem has a solution. This work has been obtained in collaboration with Marco Bonacini.

Microscopic derivation for some GENERIC systems

Manh Hong Duong (TU Eindhoven, The Netherlands)

The framework GENERIC (General Equation for Non-Equilibrium Reversible-Irreversible Coupling, Öttinger 2005) provides a systematic method to derive thermodynamically consistent evolution equations. It has been used widely in physics to model complex phenomena.

In this presentation, I will show how to derive GENERIC structures from stochastic particle systems. Two examples are discussed: the Fokker-Planck equation and the Vlasov-Fokker-Planck equation. The presentation is based on joint work with my collaborators.

Generalised geometric rigidity in mixed-growth plasticity

Janusz Ginster (Bonn, Germany)

Metals under large stresses generate dislocations, which are local defects of the crystalline lattice and can be seen as line-singularities of the elastic strain field. Straight parallel dislocations are reflected by a variational model in the orthogonal plane of the singularities. The dislocations are then point singularities of the strain β

$$\operatorname{curl}\beta = \sum_{i}^{N} b_i \delta_{x_i}$$

for some dislocation points $x_i \in \mathbb{R}^2$ and Burgers' vectors $b_i \in \mathbb{R}^2$. The energy for a strain $\beta : \mathbb{R}^2 \to \mathbb{R}^{2\times 2}$ and a corresponding singularity distribution $\mu = \sum_i^N b_i \delta_{x_i}$ is of the form

$$E_\varepsilon(\mu, \beta) = \int W(\beta),$$

where the small parameter ε describes the feasible set of singularity distributions.

Near singularities the strain field diverges as $\frac{1}{|x|}$. In order to work with a finite energy one has to either take an energy density W with quadratic growth and introduce a cut-off around singularities or choose a function W with quadratic growth for small strains and subquadratic growth at infinity. In the first case Müller, Scardia and Zeppieri proved that a strain-gradient plasticity model arises as the Γ-limit. Here the key ingredient for the compactness is a generalised rigidity estimate:

There is a rotation R s.t. $\|\beta - R\|_{L^2} \leq C\left(\|\operatorname{dist}(\beta, SO(2))\|_{L^2} + |\operatorname{curl}\beta|\right)$.

The crucial estimate in the proof is a statement first found by Bourgain and Brezis which states:

For every $f \in L^1$ the following inequality holds true

$$\|f\|_{H^{-1}} \leq C\left(\|\operatorname{div} f\|_{H^{-2}} + \|f\|_{L^1}\right).$$

In order to understand the compactness in the situation with mixed growth we aim to generalise these statements in a suitable way.

Double relaxation in line-tension energies resulting from multiple slip planes

Peter Gladbach (Bonn, Germany)

We consider an energy model for multiple parallel slip planes in an elastic medium consisting of a Peierls potential as well as a nonlocal elastic interaction both within and between the different slip planes. Besides the lattice size parameter ε, the energy features the distance h of the slip planes as an additional length scale. Under the scaling $h \sim \varepsilon^{\beta}$, $\beta \in (0,1)$, minimizers produce additional microstructure at smaller scales. This behaviour is reflected in the Γ-limit, a BV-elliptic line-tension functional, where the energy from small length scales is relaxed before being added to the large-scale energy. Since the Γ-limit must be lower semicontinuous, their sum is then relaxed again, yielding an energy density of the form $\gamma = (\gamma_{\text{short}}^{\text{rel}} + \gamma_{\text{long}})^{\text{rel}}$.

Riesz basis criterion of generalized eigenvectors and application to a nonself-adjoint Gribov operator in Bargmann space

Aref Jeribi (Sfax, Tunisia)

We are concerned with a class of unbounded perturbations of unbounded normal operators and we give a description of the changed spectrum. Moreover, we establish different conditions in terms of the spectrum to prove the existence of Riesz basis of fnite-dimensional invariant subspaces of generalized eigenvectors. The obtained results are of importance for application to a nonself-adjoint Gribov operator in Bargmann space. This is joint work with Salma Charfi and Alaeddine Damergi.

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Discrete to Continuum Limit for a 2-D Model of Water Dislocation

Leonard Kreutz (TU München, Germany)

We provide a rigorous discrete-to-continuum analysis of the two dimensional Bell-Lavis energy model for water in the low density case. By studying the Gamma-limit of proper scalings of the energy, we show emergence of so called water dislocations computing the interfacial energy between incompatible ground states. The complete description of the (asymptotic) geometry of the microscopic ground states is made quantitative by further finding an explicit formula for the energy density of the continuum limit energy functional.

An application of compactness by compensation on nonlinear parabolic type equation

Marin Mišur (Zagreb, Croatia)

We investigate conditions on two sequences $(\mathbf{u}_r)$ and $(\mathbf{v}_r)$ weakly converging to $\mathbf{u}$ and $\mathbf{v}$ in $\mathrm{L}^p(\mathbf{R}^d;\mathbf{R}^N)$ and $\mathrm{L}^q(\mathbf{R}^d;\mathbf{R}^N)$ respectively, under which the quadratic form

$$q(\mathbf{x},\mathbf{u}_r,\mathbf{v}_r) = \mathbf{Q}(\mathbf{x})\mathbf{u}_r \cdot \mathbf{v}_r$$

converges towards $q(\mathbf{x},\mathbf{u},\mathbf{v})$ in the sense of distributions. A set of sufficient conditions involves fractional derivatives and variable coefficients, both in the quadratic form and differential constraints, and they represent a generalisation of the known compactness by compensation theory. The proofs are accomplished by using recently introduced extension of H-measures – the H-distributions.

The compactness by compensation theory (pioneered by F. Murat and L. Tartar) proved to be a very useful tool in investigating problems involving partial differential equations (both linear and nonlinear). We shall present an application of the generalised concept of compactness by compensation to a nonlinear equation of parabolic type.

This is joint work with Darko Mitrović, University of Montenegro.

A variational approach to fragmentation theory

Lorenzo Nardini (SISSA, Italy)

I propose an analysis of a specific phenomenon in fracture mechanichs referred to as fragmentation. This will be studied as the asymptotic behavior for elastic bodies whose energy can be written as sum of two contributions: a bulk term, representing the elastic energy stored in the body, and a surface term, representing the dissipation spent to enlarge the crack. Summarizing I have

$$F_h(u) = h\int_\Omega f(x,\nabla u)dx + \mathcal{H}^{N-1}(J_u)\,,$$

where the last term is the area of the crack. The solution of the asymptotic problem (as h goes to infinity) will also be a solution of the minimal surface problem.

Potential theory for quasilinear elliptic equations in Sobolev spaces with variable exponent

Abdelbaset Qabil (Casablanca, Morocco)

We discuss the existence and uniqueness of solutions of the quasilinear elliptic equation in Sobolev spaces with variable exponent. These solutions are obtained by the p(.)-obstacle problem, and we investigate regularity properties of these solutions to we prove the Harnack's inequality and continuity of solutions, and we show by proving a comparison principle that Keller-Osserman property is valid and we discuss the existence of Evans functions for solutions to the quasilinear elliptic equations in Sobolev spaces with variable exponent. This is joint work with Azeddine Baalal.

References

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Variational analysis of a bidimensional model for epitaxially strained growth of islands

Emanuela Radici (Erlangen-Nürnberg, Germany)

We describe analytically a mathematical model introduced by Spencer and Tersoff for epitaxial drafting of a strained crystalline film onto a rigid substrate when a mismatch is present between the two lattices. Following outlines of a more recent work of Fonseca, Pratelli and Zwicknagl, we aim to describe self-organized surface morphologies of these films from a variational point of view. Island formation in systems such as In-GaAs/As or SiGe/Si turns out to have many technological applications: for instance, it is useful in the fabrication of modern nano-structures with optical and optoelectronic properties like quantum dot lasers. In the present work we introduce a model for a GeSi/Si system with a non trivial miscut angle, which indicates the misorientation of the substrate away from the crystal symmetry direction, and we consider only two-dimensional morphologies, which correspond to three-dimensional configurations with planar symmetry. We will report important recent results on a rigorous analysis of the 2D model for small volume islands and some regularity properties of optimal profiles.

Interfacial energies on stochastic lattices
Matthias Ruf (TU München, Germany)

We study the energy of Ising-type spin systems, when the underlying lattice of points is generated by a suitable random variable $\mathcal{L}(\omega)$. Scaling the lattice with a small paramter $\varepsilon > 0$ we are interested in the asymptotic behaviour of the corresponding energies when $\varepsilon \to 0$. Since the bulk scaling of these energies yields not enough information about the minimizing sequences we perform a higher order Γ-convergence analysis that leads to surface scalings. Under reasonable coercivity and decay assumpions on the discrete energy densities we obtain that, up to subsequences, the Γ-limit is finite only on $BV(D, \{\pm 1\})$ and admits an integral representation of the form

$$F(\omega, u) = \int_{S(u)} \phi(\omega; x, \nu_u)\, d\mathcal{H}^{n-1}.$$

If we impose more regularity on the lattice, we can make use of ergodic theory to show that the Γ-limit exists and the integrand ϕ is given by an asymptotic homogenization formula. If the generation of the lattice is ergodic, then the limit turns out to be deterministic. This is a joint work with Roberto Alicandro and Marco Cicalese (still in progress).

Analysis of a quasicontinuum method via Γ-convergence in one dimension
Mathias Schäffner (Würzburg, Germany)

Quasicontinuum (QC) methods are computational techniques to reduce the complexity of atomistic simulations in a static setting. The main idea is to couple atomistic and continuum models.

In this poster, we present some results about the analytical verification of quasicontinuum methods in the context of fracture mechanics. To this end, we start from a one-dimensional system and consider a chain of atoms with nearest and next-to-nearest neighbour interactions of Lennard-Jones type. One way to approximate the associated energy is to replace the second neighbour interaction by certain nearest neighbour potentials in a suitably chosen “continuum region”. We derive a development by Γ-convergence of this QC approximation and compare the limiting functional and its minimizers with those obtained for a fully atomistic system by Scardia, Schlömerkemper and Zanini. Joint work with Anja Schlömerkemper.

On some unbounded orbits for a class of non-autonomous Sitnikov problem

Chouhaïd Souissi (Monastir, Tunisia)

We show, using a variational method, the existence of an unbounded orbit for the non autonomous Sitnikov problem in which the potential is T-periodic for some $T > 0$, and controlled by two positive reals. The method consists in finding via a min-max argument a sequence of (kT)-periodic solutions. Then using a blow-up argument based on Ascoli-Arzela, we find an unbounded solution of our problem.

Dissipative structure of the regularity-loss type and the asymptotic stability for the Euler-Maxwell system

Yoshihiro Ueda (Kobe, Japan)

This talk is based on a joint work with Shuichi Kawashima (Kyushu University) and Shu Wang (Beijing University of Technology). In this talk, we consider the Cauchy problem of the Euler-Maxwell system in $\mathbb{R}^{\nvDash}$. The Euler-Maxwell system describes the dynamics of compressible electrons in plasma physics under the interaction of the magnetic and electric fields via the Lorentz force.

Our purpose is to study the large-time behavior of solutions to the initial value problem for the Euler-Maxwell system in $\mathbb{R}^3$. This system verifies the decay property of the regularity-loss type. Under smallness condition on the initial perturbation, we show that the solution to the problem exists globally in time and converges to the equilibrium state (or stationary solution). Moreover we derive the corresponding convergence rate of the solutions. The key to the proof of our main theorems are to derive a priori estimates of solutions by using the energy method.