



Einladung zum Oberseminar Mathematik in den Naturwissenschaften

Julius-Maximilians-Universität Würzburg
Lehrstuhl für Mathematik in den Naturwissenschaften

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Adams-Trudinger-Moser inequalities of Adimurthi-Druet type regulated by the vanishing phenomenon and its extremals¹

Let Ω be an open domain in \mathbb{R}^n ($n \geq 2$) and let $W^{m, \frac{n}{m}}(\Omega)$ with $1 \leq m < n$ be the standard higher order derivative Sobolev space in the critical exponential growth threshold. We investigate a new Adams-Adimurthi-Druet type inequality on the whole space \mathbb{R}^n which is strongly influenced by the vanishing phenomenon. Specifically, we prove

$$\sup_{\substack{u \in W^{m, \frac{n}{m}}(\mathbb{R}^n) \\ \|\nabla^m u\|_{\frac{n}{m}} + \|u\|_{\frac{n}{m}} \leq 1}} \int_{\mathbb{R}^n} \Phi_n \left(\beta \left(\frac{1 + \alpha \|u\|_{\frac{n}{m}}}{1 - \gamma \alpha \|u\|_{\frac{n}{m}}} \right)^{\frac{m}{n-m}} |u|^{\frac{n}{n-m}} \right) dx < +\infty$$

where $0 \leq \gamma < 1$, $0 \leq \alpha < 1$, $\nabla^m u$ is the m -th order gradient for u , $0 \leq \beta \leq \beta_0$, with β_0 being the Adams critical constant, and $\Phi_n(t) = e^t - \sum_{k=0}^{n-2} \frac{t^k}{k!}$. In addition, we prove that the constant β_0 is sharp. We also derive an Adams-Adimurthi-Druet inequality for a smooth bounded domain $\Omega \subset \mathbb{R}^n$ by applying our above inequality for whole space $\Omega = \mathbb{R}^n$ together with a clever scaling argument, a procedure that, to the best of our knowledge, was observed for the first time in the literature. In the subcritical case $\beta < \beta_0$, the existence and non-existence of extremal function are investigated for $n = 2m$ and attainability is proven for $n = 4$ and $m = 2$ in the critical case $\beta = \beta_0$.

¹Work in collaboration with José F. A. de Oliveira and Fábio Sodré

Ort: Mathematik Ost, 40.03.003/Zoom

Zeit: Mittwoch, 05.02.2025 um 10:30 Uhr

You are cordially invited to this lecture. The speaker will be there in person. A hybrid meeting is possible. Please request the link from anja.schloemerkerper@uni-wuerzburg.de.

gez. Anja Schlömerkerper