Collected talks with abstracts of the Workshop

Math in the Mill 2022

Contents

Thomas Bendokat (MPI Magdeburg)

Application of Differential Geometry to Numerical Model Order Reduction of Hamiltonian Systems

Abstract

We study how efficiently computable formulas for interpolation and gradient descent on the manifold of symplectic subspaces can be derived by combining Lie theory and numerical linear algebra. We then show numerical examples of how these formulas can be used in model order reduction of Hamiltonian systems defined on symplectic vectorspaces.

Matthias Frerichs (University of Würzburg)

Grothendieck Rings of Grothendieck-Verdier Categories

Abstract

TBA

David Kern (Georg-August Universität Göttingen)

Graded Geometry and Reduction

Abstract

In this talk we give a quick introduction to graded geometry, especially to 1- and 2-manifolds equipped with Q- and P-structures, and make precise what we mean with reduction of these objects. We show how this is related to Poisson reduction in the sense of Marsden-Ratiu and to infinitesimal ideal systems and give a way to extend it to graded manifolds of degree 1 and 2.

Kevin Ruck (Universität Augsburg)

Floer Homology and Powered Flybys

Abstract

The restricted three body problem (R3BP) is a physical system that occiupies mathematicians for a long time and is one of the key ingredients for successful space explorations. In recent decades we got a new powerful tool to deal with questions concerning the R3BP in the form of symplectic topology. In the first part of the talk I want to introduce the (Lagrangian) Rabinowitz-Floer homology, i.e. I will discuss the definition and also make some comments on the well-defindness. In the second part of the talk we will then see how one can use this homology to prove existance statements about orbits used to perform powered flybys.

Gregor Schaumann (University of Würzburg)

A^∞ in algebra and geometry

Abstract

In this modest introduction to the subject I want to show how to use the notion of A^∞ algebra to obtain higher operations on cohomology and how these can be useful.

Philipp Schmitt (Leibniz Universität Hannover)

Algebraic Index Theorem

Abstract

The Atiyah-Singer index theorem has an important algebraic analogue in the context of deformation quantizations, the algebraic index theorem. This theorem relates the trace on a deformed algebra with an integral of certain characteristic classes, and can be used to obtain obstructions for the existence of certain strict deformations.

In this talk, I will give an introduction to the algebraic index theorem, and outline how it can be used to prove the Atiyah-Singer index theorem.

Jonas Schnitzer (University of Freiburg)

Weak Dual Pairs in (Dirac-)Jacobi Geometry

Abstract

Adopting the omni-Lie algebroid approach to (Dirac-)Jacobi structures, we propose and investigate a notion of weak dual pairs in Dirac-Jacobi geometry. Their main motivating examples arise from the theory of multiplicative precontact structures on Lie groupoids.

In my talk I will give a short introduction to (Dirac-)Jacobi geometry, introduce the notion of weak dual pairs, explain some cases where they exist and apply this to prove a normal form theorem, which locally in special cases gives the local structure theorems by Dazord, Lichnerowicz and Marle for Jacobi structures on the one hand, and the Weinstein splitting theorem on the other hand, which are generalizations of the Darboux theorem for contact (resp. symplectic) manifolds.

Matthias Schötz (IMPAN)

Archimedean Positivstellensatz for $\mathfrak{su}(2)$

Abstract

(Non-commutative) Positivstellensätze give an algebraic description of the set of Hermitian elements of a $*$-algebra that are positive in certain well-behaved irreducible $*$-representations. In the commutative case, where the "well-behaved irreducible $*$-representations" are the complex-valued unital $*$-homomorphisms, this amounts to describing the pointwise positive elements. Somewhat surprisingly, one probably can prove Positivstellensätze for the universal envelopping algebra of compact semi-simple Lie-algebras $\mathfrak{g}$ that appear stronger than their counterparts in the commutative case, i.e. for the polynomial algebra. In this talk I will discuss the simplest special case, $g = \mathfrak{su}(2)$.

Andreas Schüssler (Radboud University Nijmegen)

Blow-ups of Lie algebroids

Abstract

In differential geometry, the real projective blow-up of a submanifold N of a smooth manifold M is a smooth manifold that arises from M by replacing N by the projectivization of the normal bundle of N in M. It comes with a canonical blow-down map mapping back to M that restricts to the fibre bundle projection on the projectivization. A similar construction can be done for Lie subalgebroids of Lie algebroids such that the blow-up is again a Lie algebroid and the blow-down map is compatible with the structure.

In my talk I will discuss the construction of the blow-up of Lie algebroids and briefly show how this might help to compute Lie algeboid cohomology.

Christiaan van de Ven (University of Würzburg)

Symmetry breaking in Nature versus Theory

Abstract

In nature, several natural phenomena occur that exhibit emergent behavior in a specific limit. Main examples include spontaneous symmetry breaking (SSB) and phase transitions: these are thought to occur only in the classical or thermodynamic limit of underlying finite quantum systems. For finite quantum systems, the ground state, playing an important role in the detection of SSB, of a generic Hamiltonian is typically unique and thus invariant under whatever symmetry group G it may be. Therefore, SSB in the usual sense of a family of asymmetric ground states connected by the action of G seems possible only in infinite quantum systems or in classical systems. Similarly, equilibrium states (or KMS states) corresponding to finite quantum (spin) systems are always unique. Consequently, phase transitions, characterized by the existence of more than one KMS state at fixed temperature, seem to occur only in infinite systems. However, in reality, both SSB and phase transitions are observed in real, and hence finite, systems. In this talk we discuss these 'paradoxical' phenomena and sketch several ideas and mechanisms that encompass both theory and reality.

Thomas Weber (University of Turin)

Quantum principal bundles and Hopf-Galois extensions

Abstract

I report on the notion of Hopf-Galois extension and its role in noncommutative differential geometry. The total space algebra is coacted upon by a Hopf algebra, with the coinvariants constituting the base algebra. Principality of the corresponding bundle is expressed by invertibility of a certain canonical map. Since in the noncommutative framework there is no „god-given" differential structure we have to impose an additional condition (in the form of a quantum Atiyah sequence) on the differential calculi we consider before calling our construct a quantum principal bundle. After discussing examples and some basic properties I prove that in the faithfully flat case quantum principal bundles amplify to graded Hopf-Galois extensions, which nicely identifies the Atiyah sequence as a higher order Hopf-Galois condition. The main result is based on a collaboration with Aschieri, Fioresi and Latini.