Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

MUΦX Lehrstuhl

Collected talks with abstracts of the Seminar

Deformation Quantization and Geometry

of the Chair for Mathematical Physics
Institute of Mathematics
University Würzburg

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1 Winter Term 2024/2025

All together

Initial Planning

18. 10. 2024, 14:00 s.t.

Abstract

Probably with cookies...

2 Summer Term 2024

Niklas Rauchenberger (JMU Würzburg)

Endpoint Geodesic Formulas for the Special Euclidean Group

12. 7. 2024, 14:00 s.t.

Abstract

The special Euclidean group SE_n is the semidirect product of the special orthogonal group SO_n and the vector space $\mathbb{R}^n$. It is a Lie group of great interest in many applications such as theoretical mechanics, robotics or computational anatomy. In this talk, the endpoint geodesic problem on the special Euclidean group will be discussed, i.e., the boundary value problem of finding geodesics that connect two given points on SE_n. We derive closed formulas for this curves that only depend on the given data by embedding the special Euclidean group into a bigger vector space. This leads to an equation involving so-called normal space involutions from which we can solve the endpoint geodesic problem explicitly.

Markus Schlarb (JMU Würzburg)

Rolling Reductive Homogeneous Spaces

5. 7. 2024, 14:00 s.t.

Abstract

In this talk, rollings of reductive homogeneous spaces are discussed from an intrinsic point of view. More precisely, if G/H is a reductive homogeneous space equipped with some invariant covariant derivative, where the reductive decomposition $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$ is fixed, rollings of $\mathfrak{m}$ over G/H are studied. In this setting, an explicit description of the configuration space as well as the distribution characterizing intrinsic rollings of $\mathfrak{m}$ over G/H can be derived. Moreover, by studying a principal fiber bundle over the configuration space equipped with a suitable principal connection, the so-called kinematic equation is obtained.

Martina Flammer (JMU Würzburg)

An invariant for persistent homology of time series

28. 6. 2024, 14:00 s.t.

Abstract

Persistent homology is one of the most popular methods in topological data analysis. Given samples from an unknown space, persistent homology estimates the structure of that space by tracking the connectivity information across several spatial scales. That information is stored in a so-called persistence module. The framework of one-parameter persistent homology is complete in the sense that one can characterize the space of isomorphism classes of persistence modules completely by an invariant that is called the barcode. Given a time varying point cloud, we assign a persistence module to it that we call an extended zigzag module. Although in this case no complete invariant exists, we define an invariant for this kind of module that visualizes some interesting features of the data. Furthermore, we provide an algorithm to compute it and discuss useful properties.

Dominik Gutwein (HU Berlin)

Calibrated submanifolds in Joyce manifolds

14. 6. 2024, 14:00 s.t.

Abstract

Calibrated submanifolds are a special class of volume-minimising submanifolds and are widely studied in differential geometry and geometric analysis. They naturally arise in manifolds with special holonomy and are sometimes viewed as their natural subobjects. This talk begins with an introduction to this topic. We will then focus on one particular class of calibrated submanifolds that appears in G_2-geometry. In particular, we will describe a construction method that produces new examples of such submanifolds inside G_2-manifolds discovered by Joyce.

Mathieu Stiénon (Penn State University)

Duflo-Kontsevich type theorem for dg manifolds

7. 6. 2024, 14:30

Abstract

Dg manifolds are a useful geometric concept which unifies many important structures including homotopy Lie algebras, foliations, and complex manifolds. In this talk, we present a Duflo-Kontsevich type theorem for dg manifolds. The Duflo theorem of Lie theory and the Kontsevich theorem regarding the Hoschschild cohomology of complex manifolds can both be derived as special cases of this Duflo–Kontsevich type theorem for dg manifolds. This is a joint work with Hsuan-Yi Liao and Ping Xu.

Ping Xu (Penn State University)

BV_∞ quantization of (-1)-shifted derived Poisson manifolds

7. 6. 2024, 13:30

Abstract

In this talk, we will give an overview of (-1)-shifted derived Poisson manifolds in the C^∞-context, and discuss the quantization problem. We describe the obstruction theory and prove that the linear (-1)-shifted derived Poisson manifold associated to any L_∞-algebroid admits a canonical BV_∞ quantization. This is a joint work with Kai Behrend and Matt Peddie.

Ana María Chaparro Castañeda (Universidade Federal Fluminense, Rio de Janeiro)

Higher Form Brackets for even Nambu-Poisson Algebras

31. 5. 2024, 14:00 s.t.

Abstract

Let k be a field of characteristic zero and $A = k[x1, ..., xn]/I with I = (f1, ..., fk)$ be an affine algebra. We study Nambu-Poisson brackets on A of arity $m \geq 2$, focusing on the case when m is even. We construct an L_∞-algebroid on the cotangent complex $\mathbb{L}_{A|k}$, generalizing previous work on the case when A is a Poisson algebra. This structure is referred to as the higher form brackets. The main tool is a P_∞-structure on a resolvent R of A. These P_∞- and L_∞-structures are merely $\mathbb{Z}_2$-graded for $m \neq 2$. We discuss several examples and propose a method to obtain new ones that we call the outer tensor product. We compare our higher form brackets with the form bracket of Vaisman. We introduce the notion of a Lie-Rinehart m-algebra, the form bracket of a Nambu-Poisson bracket of even arity being an example. We find a flat Nambu connection on the conormal module.

Michael Heins (JMU Würzburg)

Universal $\mathbb{C}$omplexifications of Lie groups

17. 5. 2024, 14:00 s.t.

Abstract

This talk gives an elementary introduction to Hochschild's notion of *universal complexification* with a focus on the resulting complex geometry. The particular setting of a vector space group provides both a motivating appetizer and ultimately reflects the infinitesimal situation of the general case. During the construction of the universal complexification, we shall meet the universal covering group, utilize Lie's seminal theorems and believe in the exactness of all good sequences. Afterwards, we develop some intuition and dispel some misconceptions by means of numerous examples, where we shall encounter some rather intriguing pathologies, all of which — how else would it be? — involve the group $\mathrm{SL}_2(\mathbb{R})$. We then discuss abstract geometric features of universal complexifications and provide some sufficient criteria to avoid the aforementioned pathologies related to the fundamental groups. If time permits, we end with some directions for ongoing and future research involving complexification of more geometrical objects. There will be one (1) complete proof and its appreciation is compulsory for all participants.

Tobias Reichel (Julius-Maximilans-Universität Würzburg)

n-fold vector bundles as associated bundles

May 10th 2024, 14:00 s.t.

Abstract

In dem Vortrag wird ein Teil meiner Masterarbeit vorgestellt. Hierfür wird zuerst die Definitionen und einige Eigenschaften von n-fachen Vektor Bündel besprochen und danach die Definitionen und Ideen von assozierten Bündeln und dem Frame Bündel von n-fachen Vektor Bündeln besprochen. Letzteres wird noch auf die Weil algebra eingegangen.

Christiaan van de Ven (JMU Würzburg)

The commutative resolvent algebra: an approach to dynamical classical lattice systems

3. 5. 2024, 14:00 s.t.

Abstract

The commutative resolvent algebra $C_R(X)$ of functions on a symplectic vector space (X, σ) is the classical analog of the celebrated resolvent algebra, introduced by Buchholz and Grundling in 2008. This commutative C^*-algebra turns out to have a rich structure, perfectly suitable to model classical dynamical systems on an infinite lattice, as well as on more general structures. We proof that for a large class of oscillating lattice systems the commutative resolvent algebra is stable under the dynamics induced by the pull-back of the ensuing Hamiltonian flow. Jointly with T. van Nuland (Technical University Delft).

Jannik Pitt (JMU Würzburg)

From Homological Algebra to the Derived Category

26. 4. 2024, 14:00 s.t.

Abstract

The basic idea of homological algebra is to replace an object X in an abelian category $\mathcal{C}$ by a resolution $X_\bullet$, which represents a generalised construction of the object. Having picked such a resolution $X_\bullet$ one then can wonder whether operations between objects X, Y, formalised by functors $F : X \to Y$, respect their resolutions $X_\bullet, Y_\bullet$. This leads to the theory of derived functors $\mathrm{R}^j F$, which measure the failure of the given operation F to respect these generalised constructions. Using these derived functors cohomology theories known from different parts of mathematics can be formulated, like Lie algebra cohomology or sheaf cohomology.

However, when passing to the derived functors one is left with only the cohomology groups thus losing information about the objects and their resolutions. The derived category $\mathcal{D}(\mathcal{C})$, introduced by Grothendieck and Verdier, is an attempt to instead work with the resolutions directly. This derived category $\mathcal{D}(\mathcal{C})$ is a powerful invariant of the original category $\mathcal{C}$, having again interpretations in different parts of mathematics.

In this talk I will try to introduce the fundamental ideas of homological algebra with the goal of understanding the idea behind the construction of the derived category.

3 Winter Term 2023/2024

Willem (Wilmer) Smilde (University of Illinois Urbana-Champaign)

Lie theory and classification of geometric structures: Bryant algebroids

February 2nd 2024, 14:00 s.t.

Abstract

In this talk, we will explore some deep connections between Lie theory and moduli problems of geometric structures. By a geometric structure we mean a coframe on a manifold. It often arises as the frame bundle of a G-structure with a connection.

Typically, a moduli problem of a coframe is given by an equation in the invariants of the coframe. Necessary conditions for the existence of solutions to the moduli problem are obtained from $d^2 = 0$: the exterior derivative must square to zero. It was already proved by Elie Cartan (under certain regularity conditions, such as analyticity and involutivity) that the condition $d^2 = 0$ is also sufficient for local existence of solutions.

The connection between "finite type" moduli problems and Lie algebroids was observed by Robert Bryant and was worked out in detail by Rui Fernandes and Ivan Struchiner. By integrating the Lie algebroid associated with the moduli problem, they obtained a complete theory of the local and global (or complete) solutions of the problem. Moreover, integration techniques can be used to construct explicit solutions to the problems, and identify precise obstructions for local solutions to extend to complete solutions.

Although many moduli problems are of finite type, it is not the typical situation. We introduce a generalization of a Lie algebroid to describe the "infinite type" moduli problems for coframes, that we call a Bryant algebroid. In many ways, it is an agglomeration of formal PDE theory and Lie algebroid theory. The aim of the project is to enable the powerful techniques in Lie theory to be used to study these types of moduli problems of coframes. I hope to explain how prolongation theory of Bryant algebroids leads to a finite rank "pro-finite" Lie algebroid, and discuss implications. Also, I will present some difficulties in identifying a global counterpart to Bryant algebroids.

This is joint work with Rui Loja Fernandes and Ori Yudilevich.

Christopher Rudolph (JMU Würzburg)

Convergent Star Products on Cotangent Bundles of Homogeneous Spaces

19. 1. 2024, 15:00 s.t.

Abstract

This talk is the sequel to my first one given in July 2023 and is about the work of my master's thesis. In their paper from April 2022, Michael Heins, Oliver Roth and Stefan Waldmann showed that the standard-ordered star product on the cotangent bundle (T^*G, ω_0) of a connected Lie group G induced by the half-commutator covariant derivative defines a continuous product on the subalgebra of entire polynomial functions endowed with a locally convex topology turning it into a Fréchet algebra. Considering the action of a closed subgroup H via right multiplication, the reduction scheme of Kowalzig, Neumaier and Pflaum yields a star product on the cotangent bundle of the quotient space G/H. On polynomial functions, this star product can be seen as a product on the quotient of H-invariant polynomial functions on G and the Poisson ideal generated by the image of a fixed basis of the Lie algebra of H under the universal momentum map via isomorphism. Now, the idea is to map a suitable quotient of the H-invariant entire polynomial functions endowed with the locally convex quotient topology injectively to this larger quotient, hopefully yielding a good subalgebra of polynomial functions on G/H together with a topology with respect to which the reduced star product becomes a continuous multiplication.

Luca Umminger (JMU Würzburg)

States in G-invariant strict Quantization

19. 1. 2024, 14:00 s.t.

Abstract

In this talk we derive the states for the star product $\star_\hbar$ of G-invariant strict quantization. Before that, we give a brief overview of the quantization of a elementary solvable symplectic

symmetric space and its C*-algebraization. In order to derive the states, we now consider the states of the Weyl-product, which result from the composition of the states of the Hilbert space $\left(\mathrm{L}^2(V),\cdot\right)$ and the positive linear mapping $S_\hbar^{\mathrm{Weyl}}:\left(\mathrm{L}^2(V),\cdot\star_\hbar^{\mathrm{Weyl}}\right)\to\left(\mathrm{L}^2(V),\cdot\right)$. Furthermore, these states can continuous linear extended to the C*-algebra. Since the star product from our quantization $\left(\star_\hbar=\tau_\hbar\circ\star_\hbar^{\mathrm{Weyl}}\circ(T_\hbar\otimes T_\hbar)\right)$ is isomorphic to the Weyl-product with isomorphism $T_\hbar$ and its inverse map $\tau_\hbar$, we derive our states for our quantization from the states for the Weyl-product.

Annika Tarnowsky

Models for differentiable stack cohomology as a generalisation of equivariant cohomology

December 1st 2023, 14:00 s.t.

Abstract

Equivariant cohomology is a cohomology theory for manifolds (or generally topological spaces) with a group action. For actions of compact Lie groups, the cohomology can be computed by the well-known Weil and Cartan models, which are given in terms of the action of the Lie algebra. Equivalently, they can be expressed in terms of the action Lie algebroid and its trivial connection. This is related to the interpretation of equivariant cohomology as the cohomology of the differentiable stack presented by the action groupoid. Generalising the Weil and Cartan models to compute the stack cohomology is a long standing open problem. I will review the problem and give an overview over the strategies and known results as well as a short insight into more recent progress on the topic.

Francesco Cattafi (JMU Würzburg)

Multiplicative frame bundle of a Lie/VB groupoid

November 24th 2023, 14:00 s.t.

Abstract

It is well known that the collection of linear frames of a smooth n-manifold M defines a principal $GL(n,\mathbb{R})$-bundle over M (called the frame bundle); more generally, this construction makes sense for any vector bundle over M. Conversely, any principal bundle together with a representation induces an associated vector bundle; these processes establish therefore a correspondence between vector bundles on one side, and principal bundles with representations on the other side.

If instead of a manifold M we begin with a Lie groupoid $\mathcal{G}\rightrightarrows M$, one can consider both the frame bundles of $\mathcal{G}$ and of M and try to "close" the resulting diagram in a natural way. The frame bundle of $\mathcal{G}$ is however too big to support a Lie groupoid structure over the frame bundle of M. In this talk, I will discuss how to fix this issue by introducing a special class of frames which interact nicely with the groupoid structure ("multiplicative frames"). At the end, I will sketch how to generalise this construction to a correspondence between VB-groupoids (groupoid objects in the category of vector bundles) and PB-groupoids (groupoid objects in the category of principal bundles). This is a joint work with Alfonso Garmendia.

Antonio Maglio (Università degli Studi di Salerno)

Shifted contact structures on differentiable stacks

November 17th 2023, 14:00 s.t.

Abstract

We propose a definition of +1-shifted contact structure on a differentiable stack, thus laying the foundations of +1-shifted contact geometry. As a side result, we will show that the kernel of a multiplicative 1-form on a Lie groupoid (might not exist as a vector bundle of Lie groupoids but) it always exists as a vector bundle of differentiable stacks and it carries a stacky version of the curvature of a distribution. Prequantum bundles over +1-shifted symplectic groupoids provide examples of +1-shifted contact structures. Time permitting, we will discuss 0-shifted contact structures which are, in some aspects, surprisingly more complicated than +1-shifted ones. This is joint work with A. G. Tortorella and L. Vitagliano.

Benedikt Pfahls (JMU Würzburg)

Constraint de Rham Cohomology

27. 10. 2023, 14:00 s.t.

Abstract

In his dissertation Marvin Dippell developed a framework called Constraint Geometry to better study the interplay of reduction and quantization of Poisson manifolds. Important objects of study in it are triples (M, C, D) of a manifold M, a closed submanifold C and a simple distribution D on C called constraint manifolds. In this presentation the focus is on expanding the geometric side of this theory, specifically we will examine the constraint de Rham cohomology of such constraint manifolds. So we will see analogues to important theorems in de Rham theory like the Mayer-Vietoris principle, and analyze the relation of constraint cohomology to other cohomology theories for the spaces M, C and the reduced space.

Matthias Frerichs (JMU Würzburg)

Grothendieck-Rings of Grothendieck Verdier Categories

20. 10. 2023, 2pm CEST/CET

Abstract

We motivate Grothendieck-Verdier duality on a category by considering categories of bimodules $\mathscr{A} - \mathscr{A} - \mathsf{Bimod}$ over an algebra $\mathscr{A}$. Here the well known notion of rigid duality can be shown to be insufficient and instead of a representation of $\mathrm{Hom}_{\mathscr{A},\mathscr{A}}(- \otimes_{\mathscr{A}} N, \mathscr{A})$ for the monoidal unit $\mathscr{A}$ and a bimodule $\mathscr{A} - \mathscr{A} - \mathsf{Bimod}$ as we would expect for a rigid category, we obtain one for $\mathrm{Hom}_{\mathscr{A},\mathscr{A}}(- \otimes_{\mathscr{A}} N, \mathscr{A}^*)$ for an algebra $\mathscr{A}^* \neq \mathscr{A}$.

In general a Grothendieck-Verdier structure on a category $\mathfrak{C}$ yields a dualizing functor $\mathsf{D}\colon \mathfrak{C} \to \mathfrak{C}$ which represents $\mathrm{Hom}_{\mathfrak{C}}(- \otimes X, K)$ for some fixed object $K \in \mathfrak{C}$. This induces a map on the Grothendieck-Ring $\mathrm{Gr}(\mathfrak{C})$.

We consider the following questions:

- What algebraic structure does a Grothendieck-Verdier duality induce on the Grothendieck-Ring?
- Given a Grothendieck-Ring with some algebraic structure and a categorification of it, when does the categorification inherit a Grothendieck-Verdier structure?

We answer both questions in the case of semisimple categories by defining the structure of a gv-based ring and give some examples of such a ring structure as well as categorifications obtained from quivers.